JOURNAL
OF
GEOMAGNETISM
AND
GEOELECTRICITY

VOL. II NO. 4

SOCIETY

OF

TERRESTRIAL MAGNETISM AND ELECTRICITY

Dec. 1950

KYOTO

JOURNAL OF GEOMAGNETISM AND GEOELECTRICITY

The fields of interest of this quarterly Journal are as follows:

Terrestrial Magnetism	Aurora and Night Sky
Atmospheric Electricity	The Ozone Layer
The Ionosphere	Physical States of the Upper Atmosphere
Radio Wave Propagation	Solar Phenomena Relating to the Above Subjects
Cosmic Rays	Electricity within the Earth

The text should be written in English, German or French. The price is provisionally set as 180 yen per copy subject to change. We hope to exchange this Journal with periodical publications of any kind in the field of natural science.

The editor.

EDITORIAL OFFICE: Society of Terrestrial Magnetism and Electricity,
Kyoto University, Japan

EDITORIAL SECRETARIES: M. OTA and S. MATSUSHITA
(Kyoto Uuiversity)

The Diurnal Variation of the Atmospheric Potential Gradient on the Summit of Mt. Fuji and along Its Slope.

By H. HATAKEYAMA & K. UCHIKAWA

Meteorological Research Institute, Tokyo, and Honjō Branch of the Aerological Observatory.

Abstract

The atmospheric potential gradient was observed on the summit of Mt. Fuji (3778m), at 5.5 gō (2800m) and Gotemba (460m), each during two or three days in August, 1947, and January, 1948. For this purpose, a polonium-collector was attached to one side of Shimizu's unifilar electrometer opposite to the side the microseope is fixed. This instrument was made by the Institute of Physical and Chemical Research of Japan, and it was so designed that it can be mounted on a tripod. The visual observation was made hourly or half-hourly to investigate the diurnal variation of the atmospheric potential gradient. According to the observational results, the ordinary semi-diurnal variation (i.e. two maxima and two minima) prevails at Gotemba but at Tarōbō the diurnal variation is not so conspicuous, whereas at 5.5 gō and at the summit the potential is greater in the daytime and smaller during the night. Thus it was made clear that the large semi-diurnal variation of the atmospheric potential gradient shows a change in character also in the vertical direction besides the local influences due to surrounding conditions.

1. Introduction.

It is well known among meteorologists that the atmospheric potential gradient observed near the earth's surface shows its maximum value in the morning and in the evening. As to the cause of the phenomenon, Whipple's theory(1) that the large value of potential gradient is in connection with the turbidity of the air near the earth's surface is the most generally accepted. While for the upper atmosphere there is no observation of the diurnal variation of the potential gradient available, except some made on aeroplane(2,3) and also by Lautner(4) who made continuous observation, though for a short period, of the potential gradient on the summit of Zugspitze (2312 m).

Investigations, in order to separate the world-wide variation from the diurnal variation of the potential gradient near the earth's surface, have been and are still

being done by atmospheric electricians. So it is very interesting to know how the type of the diurnal variation of the potential gradient changes with height. The authors of the present paper observed the diurnal change of the atmospheric potential gradient with a portable electrometer on the summit of Mt. Fuji (3778 m), at 5.5 gō (2800 m), Tarōbō (1290 m) and Gotemba (460 m), each during two or three days in August, 1947, and January, 1948, with Mr. S. Ohta[5], who made observations of the condensation nuclei.

2. Method of Observation.

The electrometer used in this observation was Shimizu's unifilar electrometer made by the Institute of Physical and Chemical Research. The metallic case of the electrometer and the microscope were removed from the stand and mounted on the top of the tripod of photograyhic camera as photographically shown in Fig. 1., in which A is the polonium-collector and B is the ambroid. The metallic case of the electrometer, the microscope, dry cells, wires are put into a small portable wooden box shown in the lower part of Fig. 2 with other accessories.

Fig. 1 Portable Electrometer A : Collector, B : Ambroid

As shown in Fig. 1, the case of the electrometer is earthed with a thin wire and its height is about 1.2 m from the earth's surface. The polonium-collector is about 3cm apart from the earthed case of the electrometer. When the observation is made on the plane earth's surface, the atmospheric potential gradient is obtained from the indication of the electrometer by multiplying the ordinary plane reduction coefficent. An example of the coefficient is shown in Table 1, which was measured at Kashiwa race-course, Chiba Prefecture, on September, 1947.

Table 1

Height of the collector (m)	Coefficient of the plane reduction	Remarks
1.16	2.02	For Tarōbō and Gotemba
0.58	4.91	For 5.5 gō
0.34	7.26	Collector on the upper corner
0.27	9.77	Collector on the lower corner

The scenes of actual observation are shown in Figs. 2 and 3. Fig. 2 shows that at Tarōbō and Fig. 3 that on the summit of Mt. Fuji. On the summit the potential gradient is so large that we can hardly catch the image of the fiber in the field of microscope.

Fig. 2 At Tarōbō

Fig. 3 On the summit of Mt. Fuji.

In the actual observation, there are growing trees in the neighbourhood of the observing spot as seen in Fig. 2, or the spot itself is a peaked summit as seen in Fig. 3. Thus another reductiou factor to reduce these topographical effects is necessary. To determine the reduction factor we have no other means than to estimate it from the position of the electrometer and the topography around it considering the equipotential surface. But we do not apply this reduction to our observations because our aim is only to clarify the type of the diurnal variation of the potential gradient.

For the observation during the night, electric torch was placed on the same side as the collector. The indication of the electrometer becomes smaller when the electric torch is brought near by. The reading of the electrometer during the night was reduced to that of the daytime by multiplying the factor duly determined.

In August, 1947, the observation was made at Tarōbō from 14h, 18th to 11h, 19th, at 5.5. gō from 7h, 20th to 8h, 21st, on the summit from 16h, 21st to 7h, 24th, at 5.5 gō again from 12h, 24th to 9h, 25th and at Tarōbō again from 14h 20m, 25th to 9h 30m, 26th. The observation was made hourly in principle, and occasionally half-hourly or bi-hourly. The observation was discontinued for several hours at night for sleeping.

In December, 1947, and January, 1948, the observation was made at Gotemba from 11h, 29th to 11h, 30th, on the summit from 17h, 2nd to 11h, 4th, at Tarōbō from 9h, 6th to 12h, 7th and at Gotemba again from 15h, 7th to 17h, 8th.

3. Results of Observation.

Results of observation in Summer is shown graphically in Fig. 4 and results in Winter in Fig. 5. The abscissa is the time and the ordinate is the potential gradient expressed in volts per meter. As already mentioned the value of the potential gradient

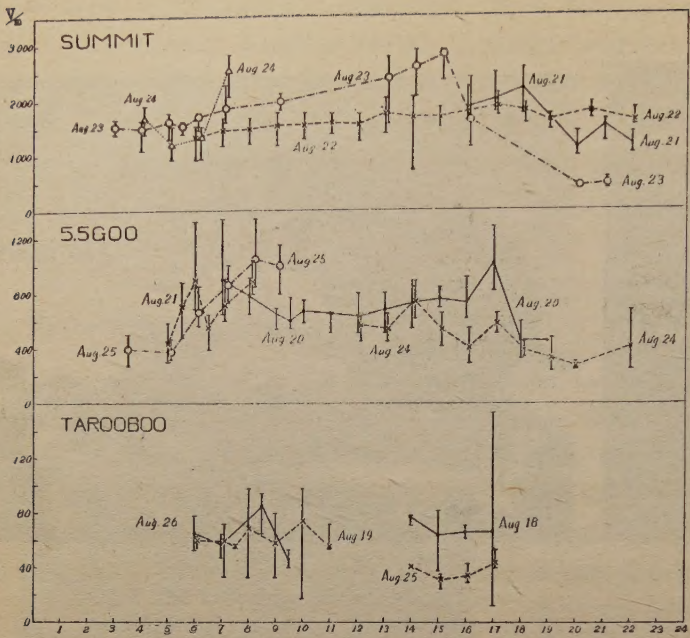
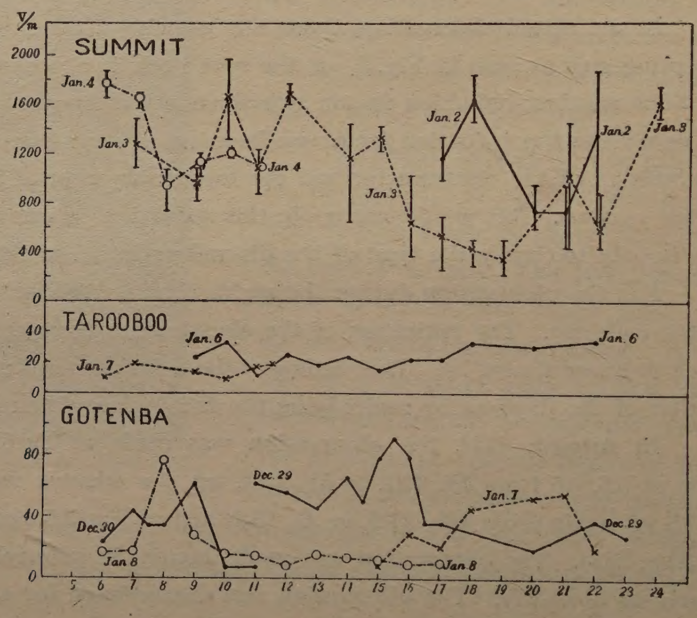


Fig. 4

Fig. 5



is not multiplied by the reduction factor due to the topography in the neighbourhood of the observing spot. The actual potential gradient at Gotemba and Tarōbō would be about twice as large as that shown in Figs. 4 and 5, and that at 5.5 gō and the summit would be far smaller than that shown in the same figures. In them, the full vertical lines show the range of the fluctuations of the observed potential gradient.

On inspecting Fig. 4 and 5 we find at once that the value of potential gradient at Gotemba and Tarōbō is the same order of magnitude as at ordinary land stations and that at 5.5 gō and the summit is extraordinarily great. Perhaps this may be due

to the pole-effect.

As to the type of the diurnal variation of the potential gradient, that at Gotemba exhibits ordinary semi-diurnal change (i.e. two maxima and two minima). One maximum is at 8h or 9h and the other is at 17h or 18h. From parallel observations of other meteorological elements, it was found that these maxima have close relation with the trailing haze-layer or the coming and going of the turbid air caused by the sudden change of the wind direction.

At Tarōbō the potential gradient is almost constant through the whole day both in Summer and in Winter.

At 5.5 gō and the summit the potential gradient is greater in the daytime than in the night, being almost twice as large. The type of diurnal variation at Zugspitze already referred to is similar to those at 5.5 gō and the summit. This type of diurnal variation is considered as universal on a mountain higher than 2000m.

Mr. S. Ohta observed the number of the condensation nuclei at the same time with our observation of the potential gradient. According to his conclusion the number of nuclei becomes smaller as the height increases and they are most numerous just after the noon and least in the early morning. Also he concluded that this diurnal variation of the number of nuclei may be due to the transportation of the nuclei by the ascending and descending currents along the slope of the mountain.

The potential gradient becomes greater (smaller) when there comes an air current containing numerous (less) condensation nuclei, consequently less (more) small ions and having less (better) electrical conductivity. These are agreeable observational results. In the free atmosphere the range of the diurnal variation of the potential gradient will be less than that at the summit or at the side of a mountain, because in the free atmosphere, the range of the diurnal variation of the number of nuclei is considered less than that on a mountain, as Mr. Ohta estimated. These points shall be verified in the future.

In conclusion the authors wish to express their warmest thanks to Mr. S. Ohta, Mr. I. Fuzimura, Chief of Mt. Fuji Weather Station, Mr. K. Takei, Chief of Gotemba Weather Station, and other members of the station for their kindness bestowed on us.

References

(1) F.J.W. Whipple; Quart. Journ. Roy. Met. Soc., **55**, 351 (1929).

(2) A. Wigand; Luftelektrische Untersuchungen bei Flugzeugaufstiegen, (1925).

(3) Ross Gunn; Journ. Appl. Phys., **19**, 481 (1948).

(4) P. Lautner; Met. Zeit., **43**, 488 (1926).

(5) S. Ohta; Journ. Met. Soc. Japan, **28**, 188 (1950).

Sudden Changes in the Atmospheric Electric Phenomena accompanying Lightning Discharges (I)*

By Minoru KAWANO

Geophysical Institute, Tokyo University

Abstract

Changes in the atmospheric electric conductivity and in the space charge accompanying lightning discharge were examined during summer in 1948–1950. It was found out that the conductivity increases remarkably at the time of lightning discharge. We might be able to say that this increase in the conductivity is attributable to an anomalous ionization by some mechanism accompanying lightning discharges.

I. Introduction

It has been found out by several investigators (e.g. 1 and 2 in References) that the marked increase in the atmospheric electric conductivity follows thunder-showers. Changes in the electric field accompanying lightning discharges were discussed in detail by Y. Tamura (3). Those in the atmospheric electric conductivity and in the space charge in the same cases, however, have scarcely been observed. In our laboratory simultaneous observations of changes in the potential gradient, the space charge, the atmospheric electric negative partial conductivity and the point discharge current, were carried out at the time of lightning discharges. In this preliminary report, the changes in the atmospheric electric conductivity and in the space charge will be dealt with in the main.

II. Apparatus

The apparatus used for the measurements of the conductivity and the space charge are schematically showh in Fig. 1 and 2. In Fig. 1, A is the Gerdien Apparatus, whose inhaling tube of air is 1.0 metre in length. The air is drown into this tube with velocity of 1.5m/sec by a fan F. The inner rod, which is insulated from the outer cylinder by ambroid S and the guard ring G, is connected to one end of a high resistance R ($2.0\times10\Omega^{12}$), the other end of which leads to the earth through a battery of 24 volts. Both ends of R are connected to an electrometer Q, by which is indicated the potential drop between them.

In Fig. 2, K is a wire cage of 1.0 metre in diameter, C a radioactive collector

* Contribution from the Division of Geomagnetism and Geoelectricity, Geophysical Institute, Tokyo University. Series II, No. 13. (1950).

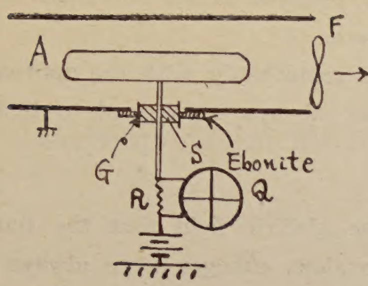


Fig. 1

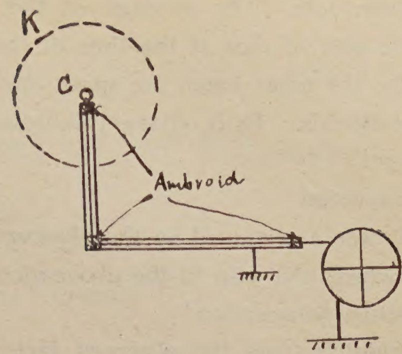


Fig. 2

connected to the quadrant electrometer. The height of the collector is 1.0 metre from the earth's surface. The linear velocity of rotation of the recording drum is 10 mm/min.

III. Results of Observation

The lightning dischages were detected by the records of the atmospheric electric field measured by the mechanical collector. The delay time of this collector is about 0.2 second, and the record obtained by it can be thought as well following to sudden changes in the atmospheric electric field. By this reason the time of occurence of lightning discharges are determined considerably acculately.

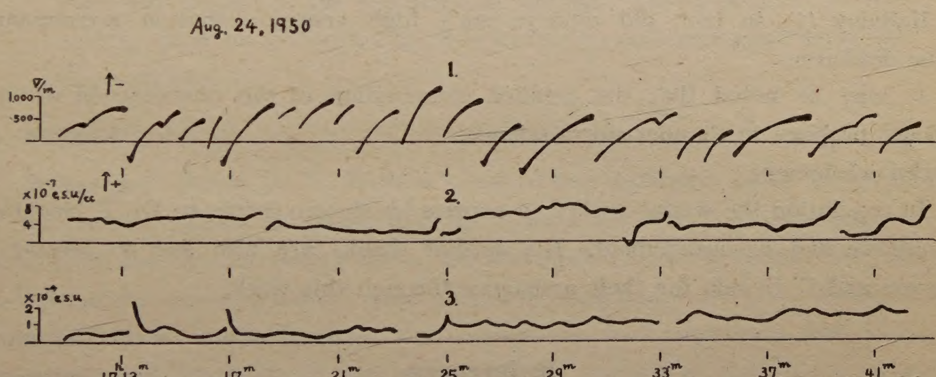


Fig. 3 1. Potential Gradient 2. Space Charge 3. Negative Conductivity

In Fig. 3 a few examples of the sudden change in the atmospheric electric negative partial conductivity are reproduced. Although the time difference between the change in the conductivity and that in the potential gradient cannot be determined directly from their photographic records. We can estimate it by tracing the image on the scale by direct vision.

The time lag measured by this method was found to be about 1–2 seconds. It is difficult, however, to improve the acculacy in determing this time lag, owing to the innertia of the needle of the electrometer. Although the distance between thunder-cloulds and the observing station did not exceed 10 km in every case, the time lag and the magnitude of sudden change in the conductivity were not constant through

all observations. The average of the anomalous increses in the conductivity was about 40–50% of that at the time of thunder-showers.

On the other hand, the space charge changes remarkably with the approach of thunder-cloulds. Each change, however, does not always correspond to lightning discharge.

IV. Discussion

It was ascertained by the observations of the electric field that the thunder-clouds which give rise to the above-mentioned anomalous changes were always those of 'positive polarization.'

Judging from the observed facts that change in the space charge accompaning lighting discharges were very small, the positive partial conductivity would probably increase simultaneously with the increase of the negative conductivity. If sudden changes above-mentioned were caused in the electric field at the time of lighting discharge, the space charge will also have to be changed, because the electric charge induced in the earth's surface should also be subject to change. The sudden change in the atmospheric electric conductivity accompanying lighting discharge, therefore, seems not to be due to tee change in the electric field.

To explain this phenomenon we must consider some anomalous ionization near the ground at the time of lightning discharge. If a high energy radiation from lightning discharge is assumed, we shall be able to explain this phenomenon reasonably. E.C. Halliday (4), in fact, did observe such high energy radiation accompanying lighting discharge.

It may be noted that the detailed examination of this phenomenon will be a significant problem in thunder-storm science.

V. Acknowledgment

In conclution the writer wishes to express his sincere thanks to Dr. T. Nagata for his guidance and encouragement. His cordial thanks are also due to Messers T. Shimazaki and E. Uchida for their assistance through this work.

References

(1) G.R. Wait and A.G. McNish: Month. Weath. Rev. **62**, 1 (1934).

(2) H. Hatakeyama: Chūo Kishōdai (Central Meteorological Observatory) Gijutsukan Yōseijo Kenkyū Hōkoku. No. 1, 1.

(3) Y. Tamura: Chikyubutsuri (Geophysics) Vol. 4 No. 3

(4) E.C. Halliday: Phys. Rev. **60**, 2 (1941).

Current System for S_D-Field and the Bay Disturbance*

By Naoshi FUKUSHIMA

Geophys. Inst., Tokyo University

Abstract

The dynamo-theory for Sq has been proved to be applicable also for S_D or the bay disturbance in the geomagnetic field. The disturbance field of S_D or bay is generally larger in the dark hemisphere compared with that in the sunlit hemisphere. This fact can be interpreted as a result of the condition that the electric conductivity of the auroral zones is markedly large, especially in the dark hemisphere. In this paper, the current system derived from the dynamo-theory with such conditions is numerically computed for various probable cases with the aid of the perturbation method. The calculated current system, such as given in Fig. 5, is in fairly good agreement with the observed one.

§ 1. Introduction

It has been shown by T. Rikitake[1] and further extended by the present author[2] that the current system corresponding to S_D-field and the bay disturbance can be derived from the dynamo-theory under the assumption that the conductivity above the so-called auroral zones is much higher compared with that in the other regions. In the calculation hitherto made, however, the conductivity in each region was assumed to be invariant with longitude as well as latitude for the sake of mathematical simplicity. Although the calculated current system well agrees, as a first approximation, with that derived from the actual analysis of the geomagnetic data, there remain some discrepancies between them; For example, the current intensity is large in the dark hemisphere compared with that in the sunlit hemisphere. Such a discrepancy might be removed by adopting the more plausible assumption on the distribution of the conductivity of the upper atmosphere. In this paper, the calculated current system under the appropriate assumption of the conductivity was compared with the observed result. The method used here is a similar one with that used by A. Schuster[3] in the case of examining Sq current system, that is, the perturbation method.

---

* Contribution from the Division of Geomagnetism and Geoelectricity, Geophysical Institute, Tokyo University. Series II, No. 12 (1950)

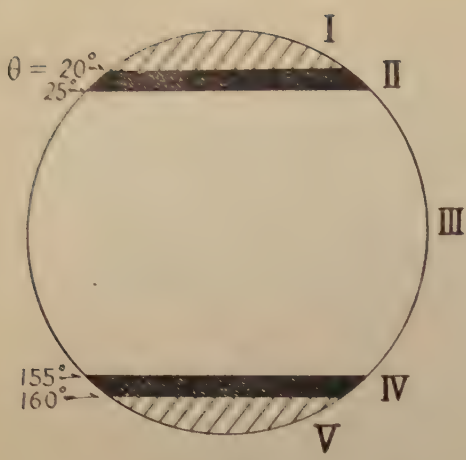


Fig. 1

§ 2. Assumptions on the distribution of the conductivity in the upper atmosphere.

As shown in Fig. 1, the conducting layer of radius R_0, which is concentric with the earth, is divided into 5 parts with respect to geomagnetic latitude, namely

the polar caps (regions I and V) ($\theta=0^\circ\sim20^\circ$, $160\sim180^\circ$),
the auroral zones (regions II and IV) ($\theta=20^\circ\sim25^\circ$, $155^\circ\sim160^\circ$),
and the equatorial zone (region III) ($\theta=25^\circ\sim155^\circ$).

In the equatorial zone, the total conductivity of the conducting layer K is assumed to be

$$K=K_0(1+\gamma'\cos\chi),$$

where K_0 is a constant, χ denotes the zenith distance of the sun, and γ' a constant less than unity. If the sun's declination is zero,

$$\cos\chi=-\sin\theta\cos\varphi$$

where φ is longitude reckoned from the midnight meridian.

As for the conductivity in the higher latitude regions, its dependence on χ could be ignored, because $\gamma'\cos\chi\ll1$. According to the result of the actual analysis of the geomagnetic data in the auroral zones, such as by M. Hasegawa[4], L. Harang[5], H.C. Silsbee and E.H. Vestine,[6] the conductivity there can be considered to be very large especially in the dark hemisphere. It will be recognized that its maximum value takes place at midnight in the first approximation. Hence, the conductivity of the auroral zones is assumed to have a form

$$K=bK_0(1+\gamma\cos\varphi),$$

where b is a constant much larger than unity, and $\gamma<1$. In the polar caps, the conductivity assumes

$$K=aK_0,$$

where a is a constant.

Summarizing the assumptions on the distribution of the conductivity in this calculation, the total conductivity in each regions is

$$\left.\begin{array}{ll} \text{in the polar caps} & K=aK_0, \\ \text{in the auroral zones} & K=bK_0(1+\gamma\cos\varphi), \\ \text{and in the equatorial zone} & K=K_0(1-\gamma'\sin\theta\cos\varphi). \end{array}\right\} \qquad (1)$$

§ 3. General theory

As in the case of the ordinary dynamo-theory, the air motion in the conducting shell of radius R_0 has a velocity $\mathbf{v}(u,v)$, which can be derived from velocity potential

ψ, and ψ is assumed to be expanded by surface spherical harmonics, namely

$$\psi=\sum_N\sum_M k_N^M P_N^M(\cos\theta)\sin(M\varphi+\alpha_N^M). \tag{2}$$

The earth's magnetic field is replaced by the magnetic field of a centred dipole of moment M, with its axis coincident with the rotation axis of the earth. Vertical component of the earth's magnetic field H_Z can be given by

$$H_Z=-\frac{2M}{R_0^3}\cos\theta\equiv-2G\cos\theta. \tag{3}$$

Then, the current function J for the stationary state must satisfy the following equations:

$$\left.\begin{aligned}&\frac{\partial J}{R_0\sin\theta\,\partial\varphi}=KvH_Z-K\frac{\partial S}{R_0\partial\theta},\\&-\frac{\partial J}{R_0\partial\theta}=-KuH_Z-K\frac{\partial S}{R_0\sin\theta\partial\varphi},\end{aligned}\right\} \tag{4}$$

where S denotes the electrostatic potential, and K is the total conductivity in each region given by the relation (1).

3. 1. Solution in the auroral zones

Substituting the expression for the conductivity in the auroral zones into (4), and neglecting the higher order terms of γ, (4) becomes

$$\left.\begin{aligned}&2bK_0G\cot\theta\frac{\partial\psi}{\partial\varphi}=bK_0\frac{\partial S}{\partial\theta}+(1-\gamma\cos\varphi)\frac{\partial J}{\sin\theta\partial\varphi},\\&-2bK_0G\cos\theta\frac{\partial\psi}{\partial\theta}=bK_0\frac{\partial S}{\sin\theta\partial\varphi}-(1-\gamma\cos\varphi)\frac{\partial J}{\partial\theta}.\end{aligned}\right\} \tag{5}$$

Let S_0 and J_0 denote the expressions for S and J when $\gamma=0$ respectively. Then, in general expression,

$$\left.\begin{aligned}&S=S_0+S', && S'=O(\gamma),\\&J=J_0+J', && J'=O(\gamma),\end{aligned}\right\} \tag{6}$$

Where S' and J' satisfy the following equations:

$$\left.\begin{aligned}&bK_0\frac{\partial S'}{\partial\theta}-\gamma\cos\varphi\frac{\partial J_0}{\sin\theta\partial\varphi}+\frac{\partial J'}{\sin\theta\partial\varphi}=0,\\&bK_0\frac{\partial S'}{\sin\theta\partial\varphi}+\gamma\cos\varphi\frac{\partial J_0}{\partial\theta}-\frac{\partial J'}{\partial\theta}=0.\end{aligned}\right\} \tag{7}$$

Eliminating S' from (7), the following equation can be derived:

$$\frac{\partial^2J'}{\sin\theta\partial\varphi^2}+\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial J'}{\partial\theta}\right)-\gamma\left\{\frac{\partial}{\partial\varphi}\left(\cos\varphi\frac{\partial J_0}{\sin\theta\partial\varphi}\right)+\cos\varphi\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial J_0}{\partial\theta}\right)\right\}=0. \tag{8}$$

On the other hand, the equation for J_0 is

$$\frac{\partial^2J_0}{\sin\theta\partial\varphi^2}+\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial J_0}{\partial\theta}\right)+2bK_0G\left\{\cot\theta\frac{\partial^2\psi}{\partial\varphi^2}+\frac{\partial}{\partial\theta}\left(\sin\theta\cos\theta\frac{\partial\psi}{\partial\theta}\right)\right\}=0, \tag{9}$$

and the solution in these regions becomes

$$\left.\begin{aligned}&J_0=-2bK_0G\sum_N\sum_M S_N^M(\theta)\sin(M\varphi+\alpha_N^M)-2bK_0G\sum_m\{C_2^m p_m(\theta)\sin(m\varphi+\alpha_{r,II}^m)+C_3^m q_m(\theta)\\&\qquad\times\sin(m\varphi+\alpha_{q,II}^m)\}\\&\qquad\qquad\qquad\text{for the northern auroral zone,}\\&J_0=-2bK_0G\sum_N\sum_M S_N^M(\theta)\sin(M\varphi+\alpha_N^M)-2bK_0G\sum_m\{C_6^m p_m(\theta)\sin(m\varphi+\alpha_{r,IV}^m)+C_7^m q_m(\theta)\\&\qquad\times\sin(m\varphi+\alpha_{q,IV}^m)\}\\&\qquad\qquad\qquad\text{for the southern auroral zone,}\end{aligned}\right\} \tag{10}$$

where

$$S_N^M(\theta)=k_N^M\left[\frac{N\{(N+1)^2-M^2\}^{\frac{1}{2}}}{(N+1)(2N+1)}P_{N+1}^M(\cos\theta)+\frac{(N+1)(N^2-M^2)^{\frac{1}{2}}}{N(2N+1)}P_{N-1}^M(\cos\theta)\right], \quad (11)$$

$$p_m(\theta)=\sin^m\theta\left[1+\frac{m(m+1)}{2!}\cos^2\theta+\cdots\right.$$
$$\left.+\frac{m(m+1)\{2(3+2m)+m(m+1)\}\cdots\{(2S-2)(2S-1+2m)+m(m+1)\}}{(2S)!}\cos^{2S}\theta+\cdots\right], \quad (12)$$

$$q_m(\theta)=\sin^m\theta\left[\cos\theta+\frac{2+2m+m(m+1)}{3!}\cos^3\theta+\cdots\right.$$
$$\left.+\frac{\{2+2m+m(m+1)\}\{3(4+2m)+m(m+1)\}\cdots\{(2S-1)(2S+2m)+m(m+1)\}}{(2S+1)!}\cos^{2S+1}\theta+\cdots\right], \quad (13)$$

and C_2^m, C_3^m, C_6^m, C_7^m, $\alpha_{p,\,II}^m$, $\alpha_{p,\,IV}^m$, $\alpha_{q,\,II}^m$ and $\alpha_{q,\,IV}^m$ are the arbitrary constants. The first term in the right hand side of (10) is the particular solution, while the remaining two terms are the general solutions. The particular solution of J_0 should be substituted into (8), and then J' can be obtained.

3. 2. Solution in the equatorial region

Substitution of the expression for the conductivity in this region into (4) gives the equations:

$$\left.\begin{aligned}2K_0G\cot\theta\frac{\partial\psi}{\partial\varphi}&=K_0\frac{\partial S}{\partial\theta}+(1+\gamma'\sin\theta\cos\varphi)\frac{\partial J}{\sin\theta\partial\varphi},\\ -2K_0G\cos\theta\frac{\partial\psi}{\partial\theta}&=K_0\frac{\partial S}{\sin\theta\partial\varphi}-(1+\gamma'\sin\theta\cos\varphi)\frac{\partial J}{\partial\theta},\end{aligned}\right\} \quad (14)$$

which correspond to (5) in the above case. After a similar treatment as before, the equations for J' in this region will be written as follows:

$$\left.\begin{aligned}K_0\frac{\partial S'}{\partial\theta}+\gamma'\cos\varphi\frac{\partial J_0}{\partial\varphi}+\frac{\partial J'}{\sin\theta\partial\varphi}&=0,\\ K_0\frac{\partial S'}{\sin\theta\partial\varphi}-\gamma'\sin\theta\cos\varphi\frac{\partial J_0}{\partial\theta}-\frac{\partial J'}{\partial\theta}&=0,\end{aligned}\right\} \quad (15)$$

or

$$\frac{\partial^2 J'}{\sin\theta\partial\varphi^2}+\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial J'}{\partial\theta}\right)+\gamma'\sin^2\theta\cos\varphi\frac{\partial^2 J_0}{\partial\theta^2}+2\gamma'\sin\theta\cos\theta\cos\varphi\frac{\partial J_0}{\partial\theta}-\gamma'\sin\varphi\frac{\partial J_0}{\partial\varphi}$$
$$+\gamma'\cos\varphi\frac{\partial^2 J_0}{\partial\varphi^2}=0. \quad (16)$$

The solution of the equation for J_0 is

$$J_0=-2K_0G\sum\sum S_N^M(\theta)\sin(M\varphi+\alpha_N^M)-2K_0G\sum\{C_4^m p_m(\theta)\sin(m\varphi+\alpha_{p,\,III}^m)$$
$$+C_5^m q_m(\theta)\sin(m\varphi+\alpha_{q,\,III}^m)\}, \quad (17)$$

where C_4^m, C_5^m, $\alpha_{p,\,III}^m$ and $\alpha_{q,\,III}^m$ are the arbitrary constants.

3. 3. Solution in the polar caps.

Since the conductivity in the polar caps assumes a constant everywhere, $J'=0$ or $J=J_0$. The differential equation for J is

$$\frac{\partial^2 J}{\sin\theta\partial\varphi^2}+\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial J}{\partial\theta}\right)+2aK_0G\left\{\cot\theta\frac{\partial^2\psi}{\partial\varphi^2}+\frac{\partial}{\partial\theta}\left(\sin\theta\cos\theta\frac{\partial\psi}{\partial\theta}\right)\right\}=0, \quad (18)$$

and the solution is

$$\left.\begin{aligned} J &= -2aK_0G\sum_N\sum_M S_N^M(\theta)\sin(M\varphi+\alpha_N^M)-2aK_0G\sum_m C_1{}^m u_m(\theta)\sin(m\varphi+\alpha_I{}^m) \\ &\qquad\qquad\qquad\qquad\qquad\qquad \text{for the northern cap,} \\ J &= -2aK_0G\sum_N\sum_M S_N^M(\theta)\sin(M\varphi+\alpha_N^M)-2aK_0G\sum_m C_8{}^m v_m(\theta)\sin(m\varphi+\alpha_V^m) \\ &\qquad\qquad\qquad\qquad\qquad\qquad \text{for the southern cap,} \end{aligned}\right\} \tag{19}$$

where

$$u_m(\theta)=\sin^m\theta\left[1-\frac{m(m+1)}{2\cdot1!(m+1)}(\cos\theta-1)+\cdots\right.$$
$$\left.+(-1)^S\frac{m(m+1)\{2+2m+m(m+1)\}\cdots\{(S-1)(S+2m)+m(m+1)\}}{2^S\cdot S!(m+S)!}(\cos\theta-1)^S+\cdots\right], \tag{20}$$

$$v_m(\theta)=u_m(\pi-\theta) \tag{21}$$

and $C_1{}^m$, $C_8{}^m$, $\alpha_I{}^m$ and $\alpha_V{}^m$ are the constants.

3. 4. Boundary Condition

The solution of the differential equation for the current function J is completely obtained in the above treatment. The groups of arbitrary constants $C_1{}^m$, $C_2{}^m$, ..., $C_8{}^m$ and phase angles in the expression for J must be determined by the conditions that the normal component of the current and the tangential component of the electric field given respectively by

$$\frac{\partial J}{R_0\sin\theta\partial\varphi},\qquad \frac{1}{K}\frac{\partial J}{R_0\partial\theta}$$

are continuous at the four boundaries of the five regions.

3. 5. Separation of Sq field and S_D field

I will be convenient to consider that the expression for the current function J consists of two parts, that is,

$$J=Jq+J_D, \tag{22}$$

where the current system deduced from Jq corresponds to Sq-field, while that from J_D causes S_D-field. Further, if Jq_0 and J_{D0} denote the expressions for Jq and J_D when $\gamma=\gamma'=0$, Jq and J_D could be considered to have the composition as follows:

$$\left.\begin{aligned} Jq &= Jq_0+\gamma' J'q\gamma' \\ J_D &= J_{D0}+\gamma' J'_{D\gamma'}+\gamma J'_{D\gamma} \end{aligned}\right\} \tag{23}$$

where $\gamma' J'q\gamma'$ and $\gamma' J'_{D\gamma'}$ the perturbation terms corresponding to $\gamma' K_0\cos\chi$, and $\gamma J'_{D\gamma}$ is that correspouding to $\gamma bK_0\cos\varphi$.

§ 4. Expmples of numerical calculation and the comparison of the results with the current system derived from the analysis of the geomagnetic data

In the actual analysis of the diurnal variation of the geomagnetic field, the term of $N=1$ and $M=1$ in the velocity potential is the largest and consequently the most important one. Hence, only the term of $N=1$, $M=1$ will be hereafter dealt with, that is

$$\psi=k_1{}^1P_1{}^1(\cos\theta)\sin(\varphi+\alpha_1{}^1). \tag{24}$$

By applying the method described in § 3, the current function J can be obtained as follows:

$$
\left.
\begin{aligned}
&\text{in region I} \quad J=-2aK_0G\,k_1^1\Bigg[\left\{\frac{P_2^1(\cos\theta)}{2\sqrt{3}}+C_1u_1(\theta)\right\}\sin(\varphi+\alpha_1^1)\\
&\qquad\qquad +\{\gamma D_1+\gamma' D_1'\}u_2(\theta)\sin(2\varphi+\alpha_1^1)+\{\gamma B_1+\gamma' B_1'\}\sin\alpha_1^1\Bigg],\\
&\text{in region II} \quad J=-2bK_0G\,k_1^1\Bigg[\left\{\frac{P_2^1(\cos\theta)}{2\sqrt{3}}+C_2p_1(\theta)+C_3q_1(\theta)\right\}\sin(\varphi+\alpha_1^1)\\
&\qquad +\left\{\gamma\left(q_1(\theta)+\frac{1}{2}\left(\frac{\pi}{2}-\theta\right)p_2(\theta)+\frac{1}{8}\sin 2\theta\right)+(\gamma D_2+\gamma' D'_2)\,p_2(\theta)\right.\\
&\qquad\qquad \left.+(\gamma D_3+\gamma' D'_3)\,q_2(\theta)\right\}\sin(2\varphi+\alpha_1^1)\\
&\qquad +\left\{\gamma\,\frac{1}{8}\sin 2\theta+(\gamma B_2+\gamma' B'_2)p_0(\theta)+(\gamma B_3+\gamma' B'_3)\,q_0(\theta)\right\}\sin\alpha_1^1\Bigg],\\
&\text{in region III} \quad J=-2K_0G\,k_1^1\Bigg[\left\{\frac{P_2^1(\cos\theta)}{2\sqrt{3}}+C_4p_1(\theta)+C_5q_1(\theta)\right\}\sin(\varphi+\alpha_1^1)\\
&\qquad +\left\{-\gamma'\left(\frac{1}{6}\sin^2\theta\cos\theta+\frac{4}{3}q_2(\theta)\right)+(\gamma D_4+\gamma' D'_4)\,p_2(\theta)+(\gamma D_5+\gamma' D'_5)q_2(\theta)\right\}\\
&\qquad\qquad \times\sin(2\varphi+\alpha_1^1)\\
&\qquad +\left\{\gamma'\frac{1}{24}(\cos 3\theta-3\cos\theta)+(\gamma B_4+\gamma' B'_4)\,p_0(\theta)+(\gamma B_5+\gamma' B'_5)q_0(\theta)\right\}\sin\alpha_1^1\Bigg],\\
&\text{in region IV} \quad J=-2bK_0G\,k_1^1\Bigg[\left\{\frac{P_2^1(\cos\theta)}{2\sqrt{3}}+C_6p_1(\theta)+C_7q_1(\theta)\right\}\sin(\varphi+\alpha_1^1)\\
&\qquad +\left\{\gamma\left(q_1(\theta)+\frac{1}{2}\left(\frac{\pi}{2}-\theta\right)p_2(\theta)+\frac{1}{8}\sin 2\theta\right)+(\gamma D_6+\gamma' D'_6)\,p_2(\theta)\right.\\
&\qquad\qquad \left.+(\gamma D_7+\gamma' D'_7)\,q_2(\theta)\right\}\sin(2\varphi+\alpha_1^1)\\
&\qquad +\left\{\gamma\cdot\frac{1}{8}\sin 2\theta+(\gamma B_6+\gamma' B'_6)p_0(\theta)+(\gamma B_7+\gamma' B'_7)q_0(\theta)\right\}\sin\alpha_1^1\Bigg],\\
&\text{in region V} \quad J=-2aK_0G\,k_1^1\Bigg[\left\{\frac{P_2^1(\cos\theta)}{2\sqrt{3}}+C_8v_1(\theta)\right\}\sin(\varphi+\alpha_1^1)\\
&\qquad +\left\{\gamma D_8+\gamma' D'_8\right\}v_2(\theta)\sin(2\varphi+\alpha_1^1)+\left\{\gamma B_8+\gamma' B'_8\right\}\sin\alpha_1^1\Bigg],
\end{aligned}
\right\}\quad(25)
$$

wherein C_i^1, C_i^0 and C_i^2 $(i=1, 2,\cdots,8)$ are replaced by C_i, $\gamma B_i+\gamma' B'_i$ and $\gamma D_i+\gamma' D'_i$ respectively, and

$$
\left.
\begin{aligned}
&u_0(\theta)=1, && u_1(\theta)=2\tan\frac{\theta}{2}, && u_2(\theta)=4\tan^2\frac{\theta}{2},\\
&p_0(\theta)=1, && p_1(\theta)=\operatorname{cosec}\theta, && p_2(\theta)=2\operatorname{cosec}^2\theta-1,\\
&q_0(\theta)=-\log\tan\frac{\theta}{2}, && q_1(\theta)=\cot\theta, && q_2(\theta)=\cot\theta\cdot\operatorname{cosec}\theta,\\
&v_0(\theta)=1, && v_1(\theta)=2\cot\frac{\theta}{2}, && v_2(\theta)=4\cot^2\frac{\theta}{2}.
\end{aligned}
\right\}\quad(26)
$$

Each elements in (23) will be written as follows:

$$
\left.
\begin{aligned}
&Jq_0=-2K_0G\,k_1^1\cdot\frac{P_2^1(\cos\theta)}{2\sqrt{3}}\sin(\varphi+\alpha_1^1),\\
&J'q_r'=-2K_0Gk_1^1\cdot\frac{1}{24}\left\{(\cos 3\theta-\cos\theta)\sin(2\varphi+\alpha_1^1)+(\cos 3\theta-3\cos\theta)\sin\alpha_1^1\right\},
\end{aligned}
\right\}\quad(27)
$$

over the whole conducting layer, and

$$
\left.
\begin{aligned}
J_{D0} &= \begin{cases}
-2a\,K_0 G\,k_1^1\left\{\dfrac{a-1}{a}\,\dfrac{P_2^1(\cos\theta)}{2\sqrt{3}}+C_1 u_1(\theta)\right\}\sin(\varphi+\alpha_1^1) & \text{in the polar cap,}\\[2mm]
-2b\,K_0 G\,k_1^1\left\{\dfrac{b-1}{b}\,\dfrac{P_2^1(\mathrm{co}\theta\,\theta)}{2\sqrt{3}}+C_2 p_1(\theta)+C_3 q_1(\theta)\right\}\sin(\varphi+\alpha_1^1) & \text{in the auroral zone,}\\[2mm]
-2K_0\,G\,k_1^1 C_5\,q_1(\theta) & \text{in the equatorial region,}
\end{cases}\\[4mm]
J'_{D\gamma} &= \begin{cases}
-2aK_0G\,k_1^1\left\{D_1 u_2(\theta)\sin(2\varphi+\alpha_1^1)+B_1\sin\alpha_1^1\right\} & \text{in the polar cap,}\\[2mm]
-2bK_0G\,k_1^1\left[\left\{q_1(\theta)+\dfrac{1}{2}\left(\dfrac{\pi}{2}-\theta\right)p_2(\theta)+\dfrac{1}{8}\sin 2\theta+D_2 p_2(\theta)+D_3 q_2(\theta)\right\}\right.\\
\qquad\left.\times\sin(2\varphi+\alpha_1^1)+\left\{\dfrac{1}{8}\sin 2\theta+B_2-B_3\log\tan\dfrac{\theta}{2}\right\}\sin\alpha_1^1\right] & \text{in the auroral zone,}\\[2mm]
-2\,K_0G\,k_1^1\left[D_5 q_2(\theta)\sin(2\varphi+\alpha_1^1)-B_5\log\tan\dfrac{\theta}{2}\cdot\sin\alpha_1^1\right] & \text{in the equatorial region,}
\end{cases}\\[4mm]
J'_{D\gamma'} &\begin{cases}
-2a\,K_0G\,k^{11}\left[\left\{D_1' u_2(\theta)-\dfrac{1}{24a}(\cos 3\theta-\cos\theta)\right\}\sin(2\,\varphi+\alpha_1^1)+\left\{B'_1-\dfrac{1}{24a}\right.\right.\\
\qquad\left.\left.\times(\cos 3\theta-3\cos\theta)\right\}\sin\alpha_1^1\right] & \text{in the polar cap,}\\[2mm]
-2b\,K_0G\,k_1^1\left[\left\{D_2' p_2(\theta)+D'_3 q_2(\theta)-\dfrac{1}{24b}(\cos 3\theta-\cos\theta)\right\}\sin(2\varphi+\alpha_1^1)\right.\\
\qquad\left.+\left\{B'_2-B'_3\log\tan\dfrac{\theta}{2}-\dfrac{1}{24b}(\cos 3\theta-3\cos\theta)\right\}\sin\alpha_1^1\right] & \text{in the auroral zone,}\\[2mm]
-2K_0Gk_1^1\left[\left(D'_5-\dfrac{4}{3}\right)q_2(\theta)\sin(2\varphi+\alpha_1^1)-B'_5\log\tan\dfrac{\theta}{2}\cdot\sin\alpha_1^1\right] & \text{in the equatorial region,}
\end{cases}
\end{aligned}
\right\}
\tag{28}
$$

in the northern hemisphere. C_4, D_4, D'_4, B_4 and B'_4 in (25) are all zero.

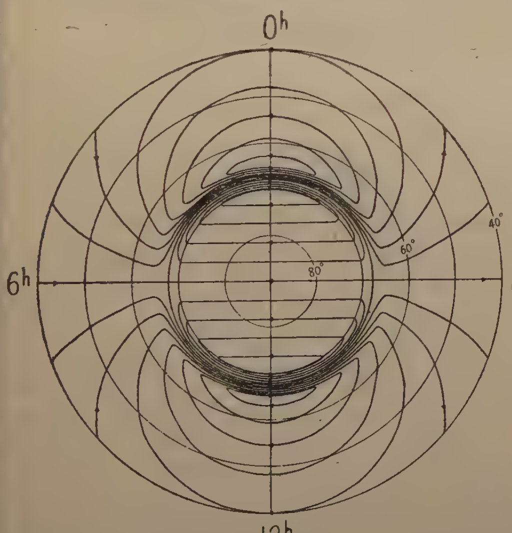


Fig. 2 The calculated current system for S_D-field or the bay disturbance under the assumption that the conductivity is assumed to be a constant in each regions.

In the numerical calculation here, K_0 is assumed to be 3×10^{-6} e.m.u., M 7.9×10^{25} c.g.s., R_0 6.47×10^8 cm, that is, the conducting layer situates 100km above the earth's surface. The value of a and b adopted here are 1 and 10 respectively. For the determination of the value of k_1^1 in ψ, the result of Chapman's analysis of the geomagnetic data in 1905(7) is referred, and α_1^1 is taken as $\frac{3}{2}\pi$(2).

In Fig. 2, the calculated current system viewed from above the north pole corresponding to S_D-field when $\gamma=\gamma'=0$, namely J_{D0} in (28) is given, where the electric current of 2.34×10^3 amp. flows between successive stream lines. As γ becomes larger, the current intensity in the auroral zone becomes more

intense in the dark hemisphere and less in the sunlit hemisphere, while the equatorial current intensity shows only a slight decrease in the dark hemisphere and increase in the sunlit hemisphere. The ratios of the total amount of the current flowing in the auroral zone and the equatorial region of the dark hemisphere to those of the sunlit hemisphere, denoted by $f(A)$ and $f(E)$ respectively, are given in Figs. 3 and 4. At that time, the current intensity in the polar caps does not show remarkable change, its change being less than 10%.

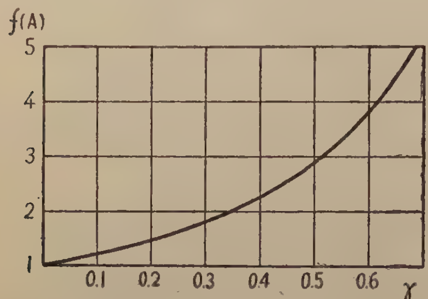


Fig. 3 Change in the ratio of the intensity of the auroral zone current in the dark hemisphere to that in the sunlit hemisphere with γ.

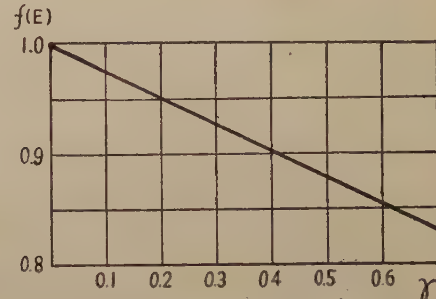


Fig. 4 Change in the ratio of the current intensity of the equatorial region in the dark hemisphere to that in the sunlit side with γ.

In Fig. 5, the current system when $\gamma=\frac{1}{2}$ and $\gamma'=0$ is shown. The current system for the bay disturbance obtained by H.C. Silsbee and E.H. Vestine[6] is also shown here for

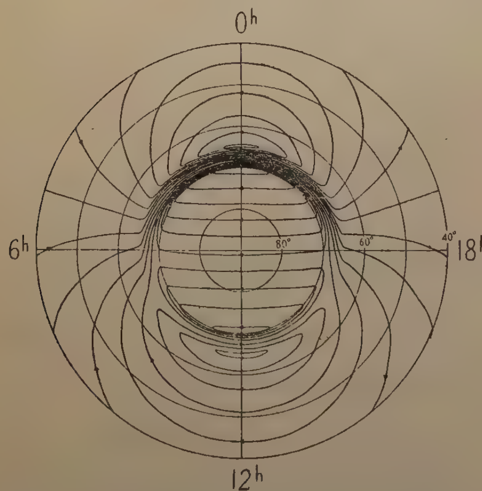


Fig. 5 The calculated current system for S_D-field and the bay disturbance when $\gamma=\frac{1}{2}$ and $\gamma'=0$.

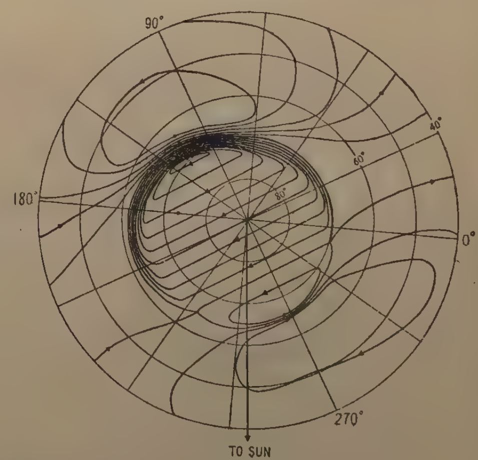


Fig. 6 The current system for the bay disturbance obtained by H.C. Silsbee and E.H. Vestine. A total of 5×10^4 amp. flows between successive current-lines.

comparison. There will be seen that the both current systems are qualitatively in good agreement with each other, although there are discrepancies of phase of about

30° and of the current intensity between them. The latter difficulty may be removed in some way, for example, by the author's hypothesis of the double conducting layers.(2)

In the analysis of the geomagnetic disturbance field at the time of magnetic storms, the Dst-component should contain, according to its definition, not only the magnetic field which is attributed to the equatorial ring current, but also the zonal part such as derived from $J'_{D\gamma}$. In other words, the zonal part is to be eliminated from the so-called S_D-field. If this zonal part is eliminated from the current system shown in Fig. 5, the current system shown in Fig. 7 can be obtained. In the latter system, the current intensity in the sunlit hemisphere and that in the dark hemisphere are nearly the same.

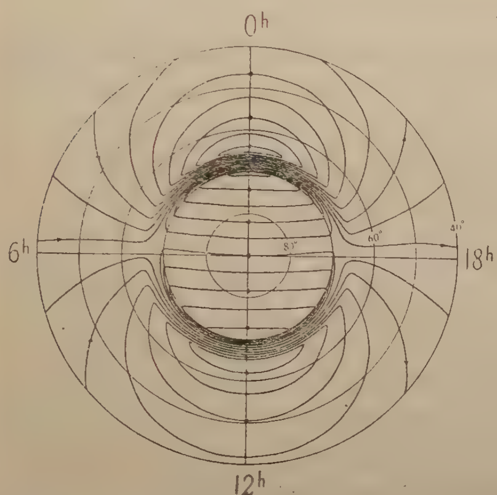


Fig. 7 The current system obtained by eliminating the zonal part from the current system in Fig. 5.

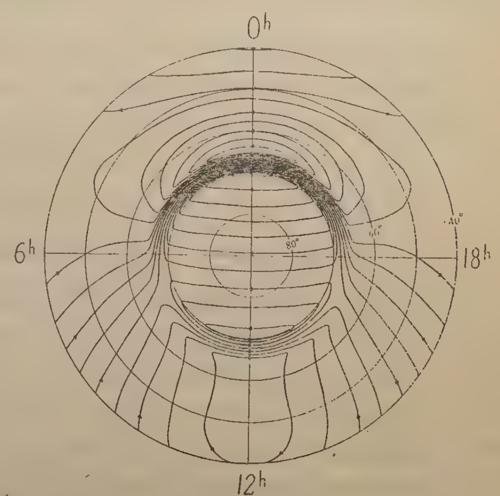


Fig. 8 The calculated current system for S_D-field when $\gamma=\frac{1}{2}$ and $\gamma'=\frac{1}{2}$.

In Fig. 8, the current system for $\gamma=\frac{1}{2}$ and $\gamma'=\frac{1}{2}$ is given. As will be seen in this figure, the current flow in the equatorial region is considerably deformed, namely it becomes very intense in the lower latitude in the sunlit hemisphere and the current vortex appears in the low latitude region in the dark hemisphere. On the other hand, the current intensity in both the auroral zones and the polar caps is scarecely affected by the value of γ'. Therefore, the asymmetry of the conductivity in the equatorial region will be not significant in the present problem.

§ 5. Conclusion.

It was shown in this study that the assumption in the dynamo-theory that the conductivity of the auroral zones is much higher, especially in the dark hemisphere, results in the production of the current system which can be identified with that derived from the analysis of geomagnetic data. In this calculation, however, the higher order terms in the harmonic expansion of the velocity potential are neglected. If these higher harmonic terms are also adopted, the resultant current system will become

more complicated, but look more like the observed system.

It must be noted here that the conductivity of the equatorial region K_0, which is assumed conventionally here 3×10^{-6} e.m.u., should be replaced by the more reliable value of about 5×10^{-8} e.m.u. obtained by T. Nagata.[8] However, we concern in this calculation under dynamo-theory only the product of the conductivity and the velocity of ionosphere wind, absolute values of conductivity being not significant. In other words, the relative magnitudes of conductivity in respective regions are to be the cause of S_D- or bay-fields under the consideration of the dynamo-theory.

At any rate, the dynamo-action is considered to be the most fundamental process to produce the various kinds of geomagnetic variation fields, although there still remains some difficulty in detail of each variation fields.

In concluding, the writer wishes to express his hearty thanks to Dr. T. Nagata for his direction throughout this study, and to Miss N. Ono for assistance in the numerical computation.

(Read: Oct. 9, 1950)

References

(1) T. Rikitake, Rep. Ionosphere Res. Japan, **2**, 57 (1949).
(2) N. Fukushima, Geophysical Notes, Tokyo Univ., Vol. 2, No. 21 (1949).
(3) A. Schuster, Phil. Trans. Roy. Soc., A, **208**, 163 (1908).
(4) M. Hasegawa, Trans. Oslo Meeting, I.A.T.M.E., 507 (1949).
(5) L. Harang, Terr. Mag., **51**, 353 (1946).
(6) H. C. Silsbee and E. H. Vestine, Terr. Mag., **47**, 195 (1942).
(7) S. Chapman, Phil. Trans. Roy. Soc. A, **218**, 1 (1919).
(8) T. Nagata, Rep. Ionosphere Res. Japan, **4**, 155 (1950).

On the Influence of the Hall Current to the Electrical Conductivity of the Ionosphere, II

By Motokazu HIRONO

Geophysical Institute, Kyoto University

Abstract

It is shown that there is a narrow region along line of zero dip near the E layer, of about 15° of latitude wide in which conductivity is very great, as a result of polarization by the Hall current. The effect of this belt is discussed by the dynamo theory, giving the result that this belt will cause enhanced diurnal variation of magnetic field near the geomagnetic equator.

1. Introduction. In the previous paper [1], hereafter referred to as I, we made a preliminary calculation on the influence of the Hall Current (in place of wich we write H.C. in the following) to the electrical conductivity of the ionosphere. The present paper describes a further examination of the problem.

T.G. Cowling [2] showed that, when there are few negative ions in the ionosphere, equ. (5) in I holds for ions and electrons separately. Therfore, mean velocity of charged particles 1 is expressed in the following equation

$$\overline{\boldsymbol{C}_1}=\frac{e_1}{m_1}\cdot\frac{\nu_1}{\nu_1^2+\omega_1^2}\boldsymbol{E}-\frac{1}{H}\frac{\omega_1^2}{\nu_1^2+\omega_1^2}\boldsymbol{h}\times\boldsymbol{E}+\frac{e_1}{m_1\nu_1}\boldsymbol{E}'' \qquad (1)$$

where $\boldsymbol{E}=\boldsymbol{E}^{(S)}+\boldsymbol{c}_0\times\boldsymbol{H},\quad \boldsymbol{h}=\boldsymbol{H}/H$

When $\boldsymbol{E}$ is perpendicular to $\boldsymbol{H}$

$$\overline{\boldsymbol{C}_1}=\frac{e_1}{m_1}\cdot\frac{\nu_1}{\nu_1^2+\omega_1^2}\boldsymbol{E}-\frac{1}{H}\frac{\omega_1^2}{\nu_1^2+\omega_1^2}\boldsymbol{h}\times\boldsymbol{E} \qquad (2)$$

Current intensity $\boldsymbol{J}$ is expressed by the equation

$$\boldsymbol{J}=\sum n_j e_j\overline{\boldsymbol{C}_j} \qquad (j=e,\ +,\ -)$$

$$=\left(\sum n_j\frac{e_j^2}{m_j}\cdot\frac{\nu_j}{\nu_j^2+\omega_j^2}\right)\cdot\boldsymbol{E}-\frac{1}{H}\left(\sum n_j e_j\frac{\omega_j^2}{\nu_j^2+\omega_j^2}\right)\cdot\boldsymbol{h}\times\boldsymbol{E} \qquad (3)$$

now let us put $\sigma^I=\sum n_j\,(e_j^2/m_j)\{\nu_j/(\nu_j^2+\omega_j^2)\}$ (4)

and $\sigma^{II}=-(1/H)\sum n_j e_j\cdot\{\omega_j^2/(v_j^2+\omega_j^2)\}$

then considering $n_e+n_-=n_+$ and $\omega^2_-/(\nu^2_-+\omega^2_-)\simeq\omega^2_+/(\nu^2_++\omega^2_+)$

we have $\sigma^{II}=(n_e e/H)\{\omega_e^2/(\nu_e^2+\omega_e^2)-\omega_i^2/(\nu_i^2+\omega_i^2)\}$, (5)

using (4) and (5), equation (3) is written as

$$\boldsymbol{J}=\sigma^I\boldsymbol{E}+\sigma^{II}\boldsymbol{h}\times\boldsymbol{E} \qquad (6)$$

Cowling [2] called σ^I and σ^{II}, direct and transverse conductivity respectively. In the following, as in I, we treat the case $\lambda<1$ near the E region. The ratio σ^{II}/σ^I with gas density n (particles/c.c.) is shown in Table 1.

Table 1

n	σ^{II}/σ^I
10^{10}	3×10^{-3}
10^{11}	0.177
10^{12}	3.0
10^{13}	17.6
10^{14}	4.1

When $\sigma^{II}\ll\sigma^I$, as in the upper part of the $F1$ layer and in the $F2$ layer ($n<10^{11}$), current flows almost in the direction of $\boldsymbol{E}$. But when $\sigma^{II}>\sigma^I$, as near the E layer ($n=6\times10^{12}$), then an impressed electromotive force $\boldsymbol{E}_0$ produces not only direct current, but also a considerable H.C., and we must consider the effect of the polarization field $\boldsymbol{E}_1$ by the latter current. Then $\boldsymbol{E}=\boldsymbol{E}_0+\boldsymbol{E}_1$, provided $\boldsymbol{E}$ is perpendicular to $\boldsymbol{H}$, therefore equation (5) becomes

$$\boldsymbol{J}=\sigma^I(\boldsymbol{E}_0+\boldsymbol{E}_1)+\sigma^{II}\boldsymbol{h}\times(\boldsymbol{E}_0+\boldsymbol{E}_1) \tag{7}$$

2. Conductivity of the E region.

Let right-handed rectangular axis o-xyz be taken, so placed that ox, oy and oz are directed to the south, east and upward respectively and unit vectors in the direction of ox, oy and oz be $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ respectively. According to the "Dynamo theory" concerning the diurnal variation of the geomagnetic field, both horizontal air velocity $\boldsymbol{c}_0$ and magnetic field $\boldsymbol{H}$, produce primarily dynamo field $\boldsymbol{c}_0\times\boldsymbol{H}=vH_z\boldsymbol{i}-uH_z\boldsymbol{j}-vH_x\boldsymbol{k}$. A. Schuster [3] in 1908 showed the possibility that $-vH_x\boldsymbol{k}$ might produce a three dimensional current system. But we think that, as the thickness of the ionosphere, even when E and F layers are combined, is sufficiently thin compared with the horizontal uniformity of the diurnal current system, $-vH_x\boldsymbol{k}$ is counteracted by the resulting electric field $E_i\boldsymbol{k}$ and $-vH_x+E_i\simeq0$. When we discuss the diurnal variation, therefore, we can eliminate both $-vH_x\boldsymbol{k}$ and $E_i\boldsymbol{k}$ from $\boldsymbol{E}$ in equation (1) and agree with Cowling's opinion [4]. In I we discussed the effect of the vertical component of the H.C., in the following we discuss effect of the horizontal component of that in addition. As in I we assume that the axes of rotation and magnetism of the earth coincide. We treat first the case, $\boldsymbol{E}_0=E_0\cdot\boldsymbol{j}$ close to the E region.

(i) Conductivity near the geomagnetic equator.

Fig. 1 shows a meridional cross section of the ionospheric E layer near the geomagnetic equator 0. In this region magnetic lines of force describe curves as AB and CD. When $\boldsymbol{E}_0$ is uniform in sufficiently wide region and $E_0>0$, H.C. tends to flow in the direction of arrows in Fig. 1, but the current will not flow in the direction counteracted by the resulting electric field, because σ^I will decrease rapidly downward in the lower part of the E layer for several ten kms in distance. In this case $\boldsymbol{E}_1=E_1\cdot\boldsymbol{n}$ where $\boldsymbol{n}=\boldsymbol{h}\times\boldsymbol{j}$; substituting this in (7) we get

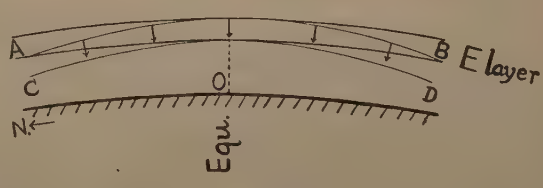


Fig. 1

$$\boldsymbol{J}=\sigma^I(E_0\cdot\boldsymbol{j}+E_1\cdot\boldsymbol{n})+\sigma^{II}(\boldsymbol{n}E_0+\boldsymbol{h}\times\boldsymbol{n}\cdot E_1) \tag{8}$$

moreover, in this case $\boldsymbol{n}\cdot\boldsymbol{J}=0$ ensues. (9)

Substituting equation (8) in the upper equation, we have
$E_1 = -(\sigma^{II}/\sigma^{I})\cdot E_0$ from this relation and (8) it follows that

$$J = \sigma^{III}\cdot E_0 \qquad \text{where} \quad \sigma^{III} = \sigma^{I}\{1+(\sigma^{II}/\sigma^{I})^2\}. \tag{10}$$

σ^{III} is considered to be resultant direct conductivity, and identical with equ. (11) in I for $\phi=0$.

In Figs. 2 and 3, σ^{III}/n_e and σ^{III}/σ^{I} are shown with n and for $\lambda<1$.

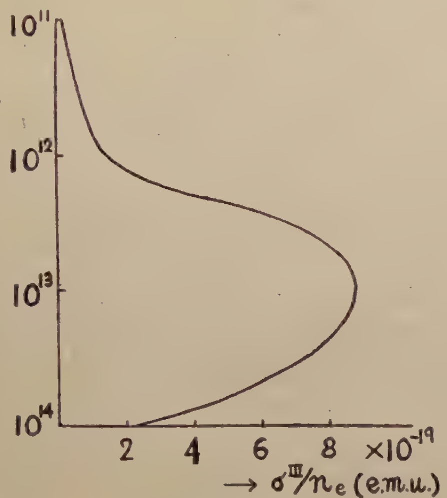


Fig. 2

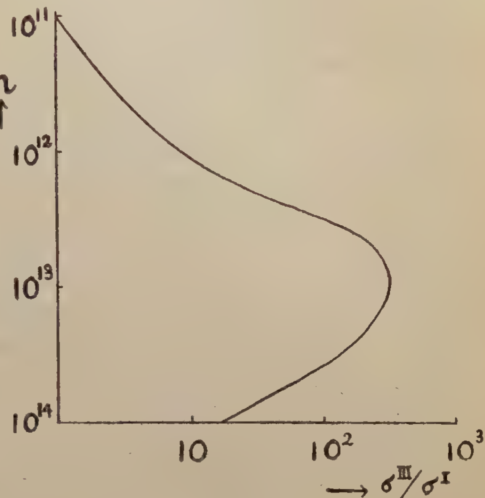


Fig. 3

From these figures it is evident that σ^{III}/n_e is maximum in the lower part of the E region.

(ii) Conductivity in middle latitudes.

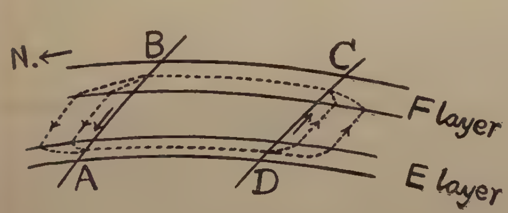


Fig. 4

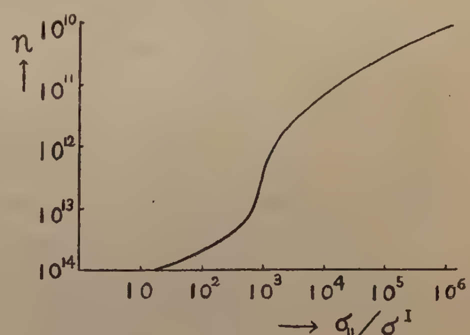


Fig. 5

Let conductivity parallel to the magnetic lines of force be $\sigma_{\parallel}$, then $\sigma_{\parallel}/\sigma^{I}$ with n is shown in Fig. 5. Fig. 4 shows that of the same kind as Fig. 1, in middle latitudes. We consider an element ABCD characterized by magnetic lines of force AB and CD. Electron density between E and F layers will be not less than one twentieth of maximum density of the E layer. When an uniform electromotive force $\boldsymbol{E}_0 = E_0\cdot \boldsymbol{j}$ ($E_0>0$), which is also uniform for longitude, is impressed between AD in the E layer, current flows not only in the direction of $\boldsymbol{j}$ but also in the direction of ADCB. Let

polarization electric field resulting from H.C. be $\boldsymbol{E}_1$ and as current between AD, we have, using equation (1)

$\boldsymbol{J}=\sigma^I(\boldsymbol{E}_0+\boldsymbol{E}_1)^{\perp}+\sigma^{II}\boldsymbol{h}\times(\boldsymbol{E}_0+\boldsymbol{E}_1)+\sigma_{\parallel}\boldsymbol{E}_1{}^{\parallel}$ where $\boldsymbol{E}_1=E_{1x}\boldsymbol{i}+(E_{1z}+E'_{1z})\boldsymbol{k}$

In I, vertical componet of H.C. and the resulting E'_{1z} were discussed and the effect of these was shown to be negligible for conductivity in higher latitudes than 10°, therefore we ignore them in the following.

Using the above equation and $\boldsymbol{k}\cdot\boldsymbol{J}=0$, we eliminate E_{1z} and get

$$\boldsymbol{J}=(\sigma^I E_{1x}/\sin^2\phi+\sigma^{II}E_0\sin\phi)\,\boldsymbol{i}+(\sigma^I E_0-\sigma^{II}E_1)\,\boldsymbol{j} \tag{11}$$

where $E_1\simeq E_{1x}/\sin\phi$

$$\therefore\quad \boldsymbol{i}\cdot\boldsymbol{J}=\sigma^I E_{1x}/\sin^2\phi+\sigma^{II}\sin\phi\cdot E_0 \tag{12}$$

where ϕ is the dip of geomagnetic field.

As current density I integrated with height in every layer, we have for AD approximately $I=\int_E\sigma^I\cdot dh\cdot E_{1x}/\sin^2\phi_m+\int_E\sigma^{II}\cdot dh\cdot E_0\sin\phi_m$, where ϕ_m is the mean value of ϕ in AD. Considering that, if $\overline{\text{AD}}$, $\overline{\text{BC}}$ are not much less than $\overline{\text{AB}}$, $\overline{\text{CD}}$, the resistances between AB and CD are far less than those between AD and BC, and that, in the F region, as $\sigma^{II}\ll\sigma^I$, even when similar $\boldsymbol{E}_0$ is impressed there, its influence to $\boldsymbol{i}\cdot\boldsymbol{J}$ is negligible, for BC we have

$$I\simeq-\int_F\sigma^I dh\cdot E_{1x}/\sin^2\phi_m$$

From above two equations $E_{1x}=a\cdot\sin^3\phi_m\cdot E_0$ (13)

where $a=-\int_E\sigma^{II}dh/(\int_E\sigma^I dh+\int_F\sigma^I\cdot dh)$

Substituting (13) in (11) and denoting mean value in the E layer by suffix E,

$$\boldsymbol{J}_E=(\sigma^I_E\cdot a+\sigma^{II}_E)\sin\phi_m\cdot E_0\boldsymbol{i}+(\sigma^I_E-\sigma^{II}_E a\sin^2\phi_m)E_0\boldsymbol{j}$$

$$\therefore\quad \boldsymbol{j}\cdot\boldsymbol{J}_E=\sigma^{IV}_E E_0\quad\text{where}\quad \sigma^{IV}_E=\sigma^I_E-a\cdot\sin^2\phi_m\cdot\sigma^{II}_E \tag{14}$$

As component of $\boldsymbol{J}$ in $\boldsymbol{i}$ flows in an opposite direction for E and F region, their effect to the magnetic field is negligible.

When $\lambda<1$, $n_{e0}=1.5\times10^5$, $\int_E\sigma^I dh=4.2\times10^{-14}\,n_{e0}=6\times10^{-9}$, $\int_E\sigma^{II}dh=1.76\times10^{-13}\cdot n_{e0}=2.64\times10^{-8}$ (e.m.u.). According to Cowling [2], in the absence of induction drag, for $F1$ layer $\int\sigma^I dh$ is 4×10^{-8}, for $F2$ layer 10^{-7}, so for $F1$ and $F2$ layers combined 1.4×10^{-7}. When BC contains $F1$ layer only $a=-0.58$, $\sigma^{IV}_E=2.28\,\sigma^I_E$ for $\phi_m=\pi/4$ and when BC contains both $F1$ and $F2$ layers $a=-0.188$, $\sigma^{IV}_E=1.4\,\sigma^I_E$ for $\phi_m=\pi/4$. Consequently σ^{IV}_E at middle latitudes will be several times of σ^I_E and we may take $\int_E\sigma^{IV}dh=10^{-8}$ as an order of magnitude. When $\boldsymbol{E}_0=\boldsymbol{E}_0\cdot\boldsymbol{i}$ we can treat the problem in a similar way, and get the same order of magnitude.

(iii) Latitude at which σ^{III} type is transformed to σ^{IV} type.

Geomagnetic lines of force are approximated by $r=a\cdot\sin^2\theta/\sin^2\theta_1$ (where $\theta=\theta_1$ at $r=a$). Let line of force which started from h km height above the equator, meet with 100km level at latitude L°, and the relation between L and h is shown in Fig. 6. h=200km ($F1$), 300km ($F2$) correspond L=7°.2, 10° respectively. At latitudes L<7°.2,

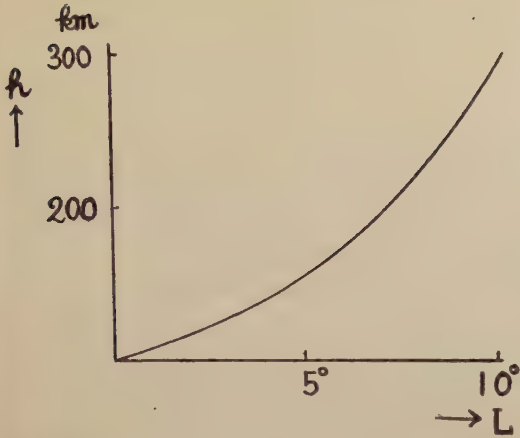


Fig. 6

lines of force, which crosses E layer, will not enter into the F region as shown in Fig. 1 therefore σ_y will not greatly differ from σ^{III}. But at latitudes $L>7.°2$, as BC of the above mentioned element ADBC enter into the F region, σ_y will decrease rapidly from $L=7.°2$ towards higher latitudes tending to $2\sigma^I$.

Up to the present we considered the effect of H.C. for an element ADCB. According to the principle of superposition, the above mentioned result will hold for the diurnal current also. When an electromotive force $E_0=(E_{0x}, E_{0y})$ is imposed, we can take $I_x=\sigma_x\cdot E_{0x}$, $I_y=\sigma_y\cdot E_{0y}$ with respect to the electric current producing the diurnal magnetic variation by reason mentioned in (ii). At the equator, for the mean value in the E layer, in which local scale height is 10km, $\sigma_{yE}\simeq\sigma_E^{III}$ and using equation (9) $\sigma_E^{III}=40\sigma^I$. As L increases, σ_{yE} will slowly decrease and beyond $L=7.°2$ σ_{yE} will rapidly decrease tending to $2\sigma^I$. Under the assumption that vertical component of H.C. does not flow, and using equ. (11) in I, we get for $(\sigma_y/\sigma^I)_E$, curve A in Fig. 7. Such an assumption will be fit, when the effective layer is sufficiently thin compared with current system. Curve A is considered to be a fairly good approximation for the above consideration, but curve B corresponding to $\sigma_{yE}=2\sigma^I$ will be better for the latitudes $L>7.°2$.

Let mean value in $0°\leqq L\leqq 7.°5$ of σ_{yE} be $\overline{\sigma_{eq}}$ and that in middle latitudes be $\overline{\sigma_m}$ then for horizontally uniform electron density we get $\overline{\sigma_{eq}}\simeq 6\overline{\sigma_m}$. Curve C is written for the case $\lambda=10$ using equ. (11) in I, for reference.

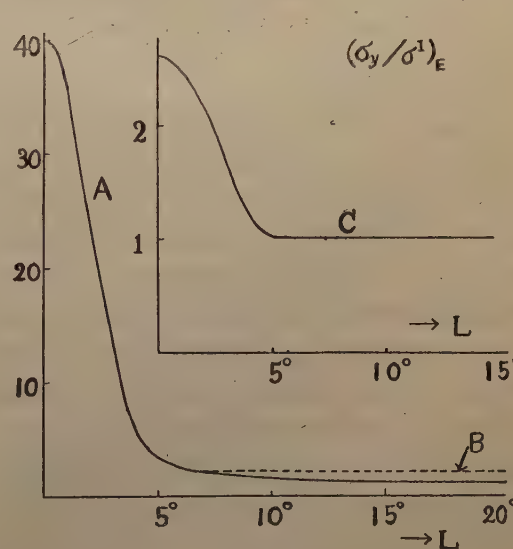


Fig. 7

3. A consideration based on the Dynamo theory.

According to the previous section, there is a narrow belt along the geomagnetic equator, of about 15° of latitude wide, in which σ_y is very great. Moreover in this belt σ_x is very great according to equ. (14) in I, and they are ohmic. By the Dynamo theory we discuss the influence of this belt to the current. Let air velocity in the spherical shell containing E region be $u=\partial\psi/\partial x=\partial\psi/a\partial\theta$, $v=\partial\psi/\partial y=\partial\psi/a\sin\theta\partial\varphi$,

current function be R, electrostatic potential be S, then we have the next relation [5],

$$\left.\begin{aligned} vH_z-\frac{\partial S}{a\partial\theta}&=\frac{1}{K}\frac{\partial R}{a\sin\theta\partial\phi} \\ -uH_z-\frac{\partial S}{a\sin\theta\partial\phi}&=-\frac{1}{K}\frac{\partial R}{a\partial\theta} \end{aligned}\right\} \tag{15}$$

where a is the radius of the shell, K is conductivity integrated with height. We assume the next distribution of K, I $K=K_1$ (const.) for $0\leqq\theta<\theta_1$, II $K=\mu K_1$ for $\theta_1<\theta<\pi-\theta_1$ III $K=K_1$ for $\pi-\theta_1<\theta\leqq\pi/2$

We take, as the velocity potential of the air, for mathematical convenience

$$\psi=k\cdot P_1^1\cdot\sin\phi \tag{16}$$

and investigate the outline of the influence of region II.

Eliminating S from (15) we have

$$\frac{\partial^2 R}{\sin\theta\partial\phi^2}+\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial R}{\partial\theta}\right)=aK\left\{\frac{\partial(vH_z)}{\partial\phi}+\frac{\partial(uH_z\sin\theta)}{\partial\theta}\right\} \tag{17}$$

We write $R=\sum\sum R_n^m$, $R_n^m=r_n^m P_n^m\sin m\phi$.

A particular integral of equation (17) in regions I, III is $R_2^1=(1/6)CK_1k\,P_2^1\sin\phi$, in region II is $R_2^1=(1/6)\,C\mu K_1k\,P_2^1\sin\phi$ (18)

where $H_z=C\cdot\cos\theta$. Next we consider the solution of the equation

$$\frac{\partial^2 R}{\sin\theta\partial\phi^2}+\frac{\partial}{\partial\theta}\left(\sin\theta\cdot\frac{\partial R}{\partial\theta}\right)=0. \tag{19}$$

Let $R=f(\theta)\cdot F(\phi)$, M^2 be constant of separation and (19) gives

$$\partial^2F/\partial\phi^2=-FM^2 \tag{20}$$

and $$\sin\theta\cdot\frac{1}{f}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right)=M^2. \tag{21}$$

In order that F be single valued function of ϕ, M must be integer and $F=\begin{matrix}\sin M\phi\\ \cos M\phi\end{matrix}$

Let two independent solution of equation (21) for M be $\psi_{1M}(\theta)$, $\psi_{2M}(\theta)$, then complete integral of (19) is

$$\sum_{M=-\infty}^{+\infty}\{\gamma_{1M}\psi_{1M}(\theta)+\gamma_{2M}\psi_{2M}(\theta)\}(\gamma_{3M}\sin M\phi+\gamma_{4M}\cos M\phi)$$

where γ_{iM} are constants. It is well known that among periodic variations of geomagnetic field, the diurnal one is the greatest, therefore for simplicity we assume that $M=1$ and take $F=\sin\phi$. In this case, bounded solutions of equation (19) in every region are [6] I $\{C_1\sin\theta/(1+\cos\theta)\}\cdot\sin\phi$

II $(C_2\cot\theta+C_3\operatorname{cosec}\theta)\cdot\sin\phi$, III $\{C_4\sin\theta/(1-\cos\theta)\}\cdot\sin\phi$.

Therefore a solution of (17) which is bounded in every region, is obtained by summing (18) and above functions in every region respectively. For this solution by the symmetry of current against equator $C_3=0$, $C_4=C_1$. Considering the continuity of normal component of current and tangential component of electric field along the boundary between I, II and II, III we can determine C_1 and C_2, then

$$\left.\begin{aligned} &\text{I} \quad R=\frac{1}{6}CK_1k\left\{P_2^1(\theta)+\frac{(\mu-1)(1+\cos\theta_1)}{\sin\theta_1(1+\mu\cos\theta_1)}P_2^1(\theta_1)\frac{\sin\theta}{1+\cos\theta}\right\}\sin\phi \\ &\text{II} \quad R=\frac{\mu}{6}CK_1k\left\{P_2^1(\theta)-\frac{(\mu-1)\sin\theta_1}{1+\mu\cos\theta_1}P_2^1(\theta_1)\cot\theta\right\}\sin\phi \end{aligned}\right\} \tag{22}$$

From above equations we can determine eastward current V,

$V = -\frac{\partial R}{a\partial\theta} = V_0(\theta)\cdot\sin\phi.$ $V_0(\theta)$ is represented in Fig. 8 in unit of $(1/6)\,CK_1 k$ for various values of θ_1 and μ.

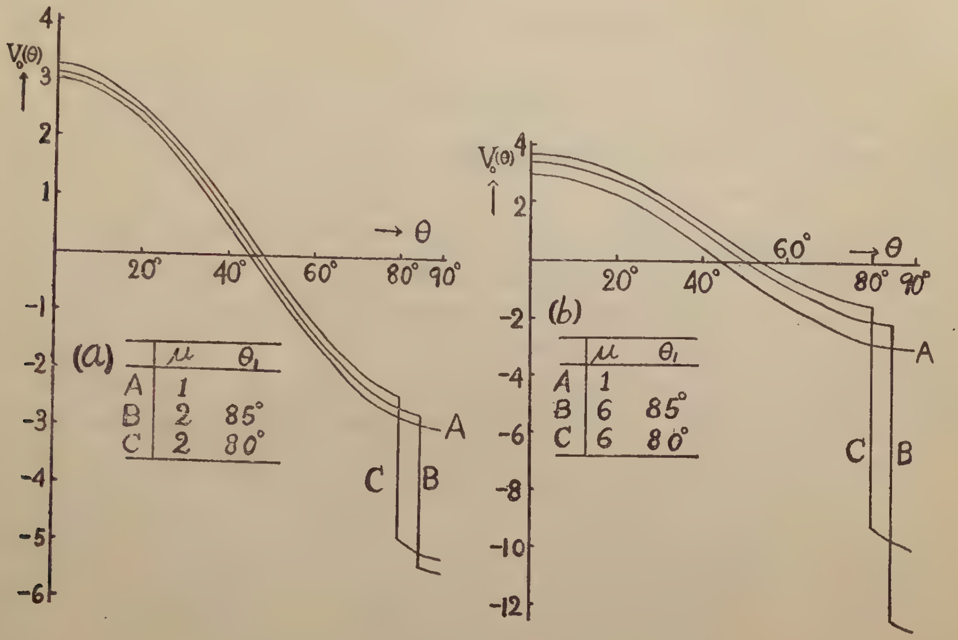


Fig. 8

The more μ increases, the more total amount of current increases, and the more current focus shifts towards equator and the more the current in lower latitudes concentrates towards equator. But, as it was shown by D.F. Martyn that some diurnal currents flow in $F1$ and $F2$ layers [7], it is impossible to determine μ directly from this result. Nevertheles the value $2 \leq \mu \leq 6$ might be appropriate.

4. Discussion of the result.

It was shown that there is a narrow region along the geomagnetic equator, of about 15° latitude wide, in which conductivity is very great, as a result of polarization by the H.C., and this belt concentrates some part of diurnal electric current, giving enhanced diurnal variation of geomagnetic field. In reality, however, the axes of rotation and magnetism do not coincide and there exist fair local anomalies of magnetic field. In the real ionosphere the high conductive belt will lie near the E layer, along the line of zero dip at that level. According to precise statiatical reseach by Prof. M. Hasegawa and Dr. M. Ota [8], a belt of enhanced geomagnetic diurnal variation lies approximately along the geomagnetic equator, but a little discrepancy exists between them. Investigation about this discrepancy will be reported in a later paper.

Acknowledgement

The writer wishes to express his hearty thanks to Prof. M. Hasegawa for his

encouragement in this work, and to Dr. S. Hayami for his advice. It is a deep pleasure to record here a debt of gratitude to Prof. T.G- Cowling for his many valuable discussions with respect to part I of this work and to Dr. D.F. Martyn for his valuable suggestions.

Many thanks are also due to Mr. H. Maeda for his assistance in this work.

References

[1] M. Hirono, J. of Geomag. and Geoele. **2**, 1, (1950).

[2] T.G. Cowling, Proc. Roy. Soc. Lon. A, **183**, 453, (1945).

[3] A. Schuster, Phil. Trans. Roy. Soc. Lon. A, **208**, 185 (1908).

[4] T.G. Cowling, Private communication. (Mar. 1951).

[5] S. Chapman and J. Bartels, 'Geomagnetism' II, 768, (1940).

[6] T. Rikitake, Rep. Ionos. Res. Japan, **2**, 58 (1948).

[7] D.F. Martyn, Proc. Roy. Soc. Lon. A. **194**, 445, (1948).

[8] M. Hasegawa and M. Ota, 'Transactions of Oslo meeting Aug. 19–28, 1948,' p. 426, (1950).

LETTER TO EDITOR

On the Analysis of Sudden Indrease of Cosmic Ray associated with Solar Flare

The sudden increase of cosmic ray intensity associated with the solar flare is of very interest because it seems to give some clue to the origin of usual cosmic rays.

The questions which arise from this phenomenaon may be divided in the following three parts:

(1) Did what kind of incoming particles give rise such unusual increase of cosmic rays?

(2) Is there any possible mechanism that can accelerate the charged particle up to several Bev at the time of solar flare?

(3) If such mechanism is possible, can such particles escape from the strong magnetic field of the sun?

Questions (2) and (3) have already been studied and answered affirmatively by Swann[1] and Forbush et al[2] respectively.

In order to study the feature and the energy distribution of the incoming rays which produced such increase, the analysis was made using the data of the increase on Feb. 28, and Mar. 7. 1942, and the following results were obtained. (The available data for this analysis were obtained at Huancayo (1°Sm), Christchurch (48°Sm), Cheltenham, (50°Nm) Godhavn (78°Nm)[3] and Tokyo (25°Nm)).

(1) The increase at Godhavn was larger than that of Cheltenltenham. The difference was surely caused by the particles with energy less than the magnetic cut off energy at Cheltenham. Furthermore, the particles must have at least 2 Bev per nucleon to penetrate through the atmosphere. From Table I the platicles satisfying

Table. 1. Magnetic cut off energy per nucleon at Cheltenham.

particle	electron	proton	α particle	other nuclei
Magnetic cut off energy/nucleon	$\approx$ 4 BeV	$\approx$ 3 BeV	$\approx$ 1.2 BeV	$\approx$ 1 BeV

above conditions are protons or electons. However, the possibility that incident particles were only electrons can be excluded from the fact that the increase was also observed under the thick (10 cm or more) lead absorber[4]. We thus conclude that almost of alli ncident particles were protons. This conclusion was confirmed by recent experiment of photographic plate carrid out by Schein[5] et al, who found that only protons were increased at the time of solar flare.

If the particles had been accelerated so gradually that the heavy particles had not been destroyed, they would have been accelerated and observed. From this argument it may be concluded that the particles were not accelerated so gradually.

(2) From the latitude effect of the increase, the energy spectra of incoming particles which produced such increase are roughly estimated.[6]. As shown in Fig. 1, the average energy of particles decreases with increasing time on Feb. 28, while it is almost constant during the increase on Mar. 7.[7]

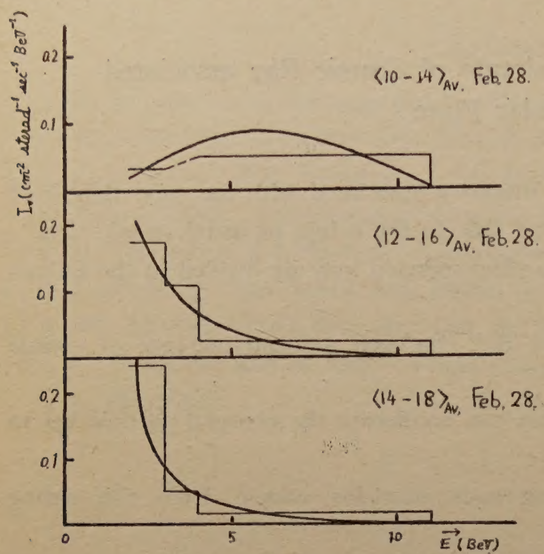


Fig. 1. Differential Spectra of Incoming Particles. <10—14>AV. means the average value from. 10 to 14 G.M.T.

This contradiction may be explained by the following assumptions :

(1) The tunnels[2] through the Störmers forbidden region opened up during the acceleration on both days. While the acceleration mechanism was gradually demped with time on Feb. 28, but was suddenly damped on Mar. 7 after having accelerated the particles for a short time.

(2) The acceleration mechanism was gradually damped on both days. While the tunnel opened up during the acceleration on Feb. 28, but on Mar. 7, it opened up for a short time and shut off before the acceleration mechanism was damped.

The assumption (1) does not seem to be plausible, and it may be due to the assumption (2). However, for the lack of information available to us, we can not decide whether this conclusion is right or not.

we wish to thank Dr. Nishina, Dr. Sekido and Mr. Miyazaki for their many helpful discussions.

By J. Nishimura

Scientific Research Inst. Ltd., Tokyo*

* Now Department Physics, Kobe University, Kabe, Japan.

(1) W.F.G. Swann, Phys. Rev. **43**, 217 (1933).
(2) S.E. Forbush, P.S. Gill. and M.S. Vallarta, Rev. Mod. Phys. **21**, 44 (49).
(3) I. Lunge, and S.E. Forbush, Terr. Mag. **47**, 185 (1942).
(4) in this issue.
(5) J.J. Lord, A.W. Elston, and M. Schein Phys. Rev. **79**, 540 (1950).
(6) T.H. Johnson, Rev. Mod. Phys. **10**, 193 (1938).
(7) This is not shown in Fig. 1, but it is evident from the data of reference (1).

昭和26年4月25日 印刷
昭和26年4月30日 發行
第2巻第4號 定價150圓
(國外定價180圓)

編輯兼發行者 日本地球電氣磁氣學會
代表者 長谷川万吉

印刷者 京都市下京區上鳥羽學校前
田中幾治郎

賣捌所 丸善株式會社京都支店
丸善株式會社 東京・大阪・名古屋・仙台・福岡

JOURNAL OF GEOMAGNETISM AND GEOELECTRICITY

Vol. II No. 4

1 9 5 0

CONTENTS